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Anisotropy, pinning, and the mixed-state Hall effect

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We have studied the mixed-state Hall effect of the high-T_c superconductors $Nd_{1.85}Ce_{0.15}CuO_4$ (NCCO), $Tl_2Ba_2CaCu_2O_8$ (Tl2212), and $YBa_2Cu_3O_7$ (YBCO), and the isotropic low-T_c superconductor amorphous Mo_3Si (a-Mo_3Si). We demonstrate the pinning independence of the Hall conductivity σ_{xy} and its consequent scaling in terms of the anisotropy of NCCO, Tl2212, and YBCO. In YBCO and a-Mo_3Si the Hall angle is enhanced as we reduce the effective pinning, yet σ_{xy} is unchanged. For all of these materials there is a vortex contribution $\sigma_{xy}\sim 1/H$ at low fields while at high fields $\sigma_{xy}\sim H$. These results provide evidence that beneath the effects of pinning and anisotropy a relatively simple and universal behavior of the mixed-state Hall effect exists.

The transport properties of superconductors in the vortex state, though widely studied, still present many puzzles. In particular, a wide range of both high-T_c and low-T_c materials show a sign change of the Hall effect below T_c.[1] In the mixed state, vortices moving with velocity **v** generate a spatially averaged electric field according to Josephson's relation $\mathbf{E}=-\mathbf{v}\times\mathbf{B}$.[2] When the flux lines are pinned a transport current flows around the vortices with no energy loss ($\mathbf{E}=0$). However, when the vortices move current passes through the vortex core, dissipating energy and generating a Hall voltage. In the simplest models this leads to a mixed-state Hall effect of the same sign as in the normal state.[3,4]

A difficulty in developing a model of the mixed-state Hall effect, in addition to predicting a sign change, is that a wide variety of temperature and field dependences have been reported,[1] at least partly due to the range of pinning strengths and anisotropy in these materials. In fact, recent models have suggested the sign change is related to the specific properties of flux pinning[5] or layered structures.[6,7] In this report, we account for anisotropy and demonstrate that the Hall conductivity σ_{xy} is *independent of pinning*, strongly suggesting that neither pinning nor anisotropic structures are the origin of the sign anomaly. In addition, the Hall conductivity for a variety of superconductors, both anisotropic high T_c and isotropic low T_c, has a contribution $\sigma_{xy}\sim 1/H$ which is presumably intrinsic to vortex dynamics.[4,8,9]

We measure the longitudinal voltage V_{xx} and the Hall voltage V_{xy} with the transport current in the ab plane perpendicular to the magnetic field (up to 9 T). The magnetic field is ramped from high to low fields at both polarities with the temperature fixed; V_{xy} is the component of the transverse voltage odd in applied field. The sample mount has a rotating stage for changing the field direction, a magnetic-field-insensitive Cernox metal film thermometer, and a carbon glass thermometer with a calibration in magnetic field. The $Tl_2Ba_2CaCu_2O_8$ (Tl2212) sample is a c-axis oriented 5000-Å-thick film prepared by laser ablation from a single target followed by post-deposition heat processing with $T_c\cong 104$ K and $j_c\approx 10^6$A/cm^2 at 77 K. The $Nd_{1.85}Ce_{0.15}CuO_4$ (NCCO) measurements were made on a c-axis oriented 20-μm-thick single crystal with $T_c\cong 24$ K (details of the preparation are given in Ref. 10). The $YBa_2Cu_3O_7$ (YBCO) measurements were made on a c-axis oriented 1000-Å-thick film grown by pulsed laser ablation with $T_c\cong 92$ K. The amorphous Mo_3Si (a-Mo_3Si) sample is an isotropic amorphous film of 250-Å thickness with $T_c\cong 7.5$ K; details of the sample preparation and characterization are given in Refs. 11 and 12.

An interesting way to vary the transport properties of anisotropic materials is to change the magnetic field direction.[13] For our measurements we rotate the sample, varying the angle θ between the c axis and the field, while keeping the current and field perpendicular to each other. In Fig. 1(a) we show the in-plane longitudinal resistivity, ρ_{xx} vs H, measured on our NCCO single crystal at $T=16$ K for $\theta=0°$, 30°, 50°, and 70°. As θ increases the ρ_{xx} vs H transitions move out to higher fields, reflecting the increase in H_{c2} as the field is directed into the ab plane. The effective mass anisotropy ratio is defined as $\Gamma\equiv m_c/m_{ab}=(H_{c2}^{\parallel}/H_{c2}^{\perp})^2=\xi_{ab}^2/\xi_c^2\approx 10^3$ for NCCO,[14] where $H_{c2}^{\parallel}$ ($\propto 1/\xi_{ab}\xi_c$) and $H_{c2}^{\perp}$ ($\propto 1/\xi_{ab}^2$) correspond to H_{c2} with the field applied parallel and perpendicular to the ab plane, respectively. In Fig. 1(b) we plot ρ_{xx} vs $H\cos\theta$. The data for all θ collapse very nicely to a single curve, indicating that only the field component parallel to the c axis ($H\cos\theta$) is contributing to the dissipation. We have observed the scaling of ρ_{xx} with $H\cos\theta$ from $T=19$ K down to $T=4$ K, which is the temperature range of our measurements. We have not pursued the high angle regime, $\theta\rightarrow 90°$, where various two-dimensional[15] (2D) and anisotropic 3D models[16,17] are distinguishable, as done on Bi2212,[18] for example. Our

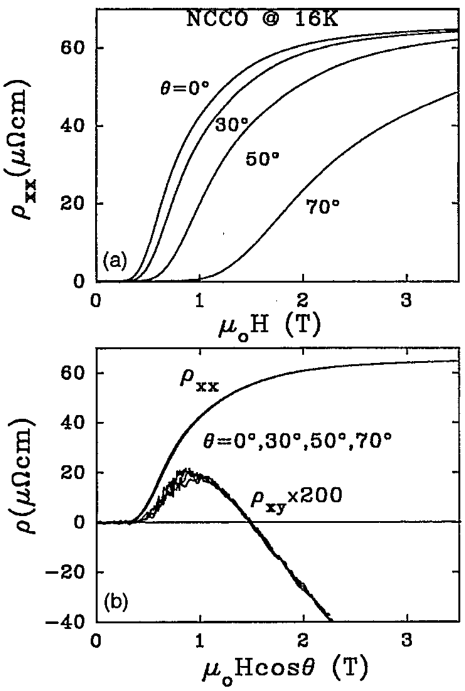

FIG. 1. (a) Longitudinal resistivity ρ_{xx} vs H at $T=16$ K, for several tilts of the field in a single crystal of $Nd_{1.85}Ce_{0.15}CuO_4$ (NCCO) with $T_c \cong 24$ K. The angle θ is measured with respect to the c axis. (b) ρ_{xx} and ρ_{xy} vs $H\cos\theta$ (the field component along the c axis).

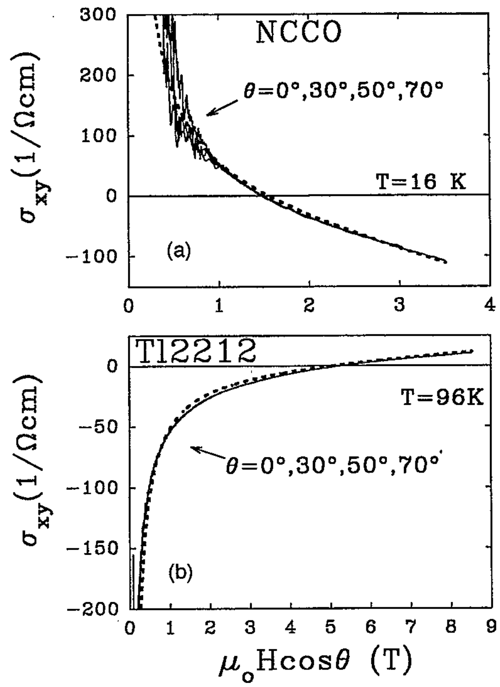

FIG. 2. (a) Hall conductivity $\sigma_{xy}=\rho_{xy}/(\rho_{xx}^2+\rho_{xy}^2)$ vs $H\cos\theta$ for NCCO single crystal at $T=16$ K (solid curves), and a fit to the data which gives $\sigma_{xy}=95$ (T/Ω cm)/$\mu_0 H_z-40$ (1/T Ω cm)$\mu_0 H_z$ (dashed curve). (b) Hall conductivity σ_{xy} vs $H\cos\theta$ for a Tl2212 film at $T=96$ K ($T_c \cong 104$ K) (solid curves), and a fit to the data which gives $\sigma_{xy}=-54$ (T/Ω cm)/$\mu_0 H_z+2$ (1/T Ω cm)$\mu_0 H_z$ (dashed curve).

primary interest is to point out that the field component along the c axis is dictating the vortex dynamics.

Also shown in Fig. 1(b) is the Hall resistivity ρ_{xy} vs $H\cos\theta$ for $T=16$ K. The scaling with $H\cos\theta$ indicates that the in-plane Hall effect, and in particular the sign anomaly, arises strictly from vortex segments aligned parallel to the c axis. In the normal state ρ_{xy} scales with $H\cos\theta$ for all materials, whereas in the mixed state of a superconductor this simple scaling form is only expected to hold in the large anisotropy limit.[16,17] In the limit of large anisotropy (and θ not too near 90°), 3D anisotropic-mass models predict $F_{ij}(H,\theta)=F_{ij}(H\cos\theta)$, where F is a transport quantity such as resistivity ρ or conductivity σ and $ij=xx$ or $ij=xy$.

As can be seen in Fig. 1, both ρ_{xx} and ρ_{xy} become immeasurably small below a minimum field $\mu_0 H_{\rm min}\approx\frac{1}{2}$ T due to flux pinning. To observe free-flux flow at low currents it is necessary to go into a regime where the pinning potential is weak ($H \gg H_{\rm min}$). However, $H_{\rm min}(T)$ is the same order of magnitude as $H_{c2}(T)$, even in relatively low-pinning materials. High-current densities can be used to suppress pinning,[12,19] but it is difficult to increase the current density without heating in single crystals. An alternative is to measure a pinning-independent quantity, such as predicted for σ_{xy},[20,21] which still elucidates the essential physics.

In Fig. 2(a) we replot the data from Fig. 1 as σ_{xy} vs H_z (σ_{xy} vs $H\cos\theta$), where $\sigma_{xy}=\rho_{xy}/(\rho_{xx}^2+\rho_{xy}^2)$. The dashed curve is a fit to $\sigma_{xy}=c_1/H_z+c_2 H_z$, yielding $\sigma_{xy}=95$ (T/Ω cm)/$\mu_0 H_z-40$ (1/T Ω cm)$\mu_0 H_z$. The linear contribution to σ_{xy} has the same sign and field dependence as the normal-state Hall conductivity. The coefficient of the $1/H$ term is opposite in sign to the normal state, thus this term leads to a sign reversal. Several models predict a vortex contribution to the Hall conductivity of the form $\sigma_{xy}\sim 1/B$;[4,8,9] we note in our experiments $B \cong \mu_0 H$. Nozières and Vinen[4] suggest this form, but with a coefficient of the same sign as in the normal state. Based on time-dependent Ginzburg-Landau (TDGL) theory, Dorsey[8] and Kopnin[9] argue that for $H \ll H_{c2}$ $\sigma_{xy}\sim 1/B$, as flux flow dominates the conductivity, and allow a sign change depending on electronic structure. They go on to argue that near H_{c2} the conductivity is the normal-state conductivity plus a correction due to vortex motion which decreases as $H_{c2}-B$.[8,9] A convenient way to interpolate between the low- and high-field regimes is to write $\sigma_{xy}=c_1/B+c_2 B$, which models our data reasonably well, but we note that this additive form for σ_{xy} is not a consequence of TDGL theory.

We observe similar results for Tl2212 [$\Gamma \approx 10^4$ (Ref. 22)] both in the angular dependence as well as the field dependence of the Hall conductivity. Displayed in Fig. 2(b) is σ_{xy} vs $H\cos\theta$ for Tl2212 at $T=96$ K and $\theta=0°$, 30°, 50°, and 70°. The dashed curve is given by $\sigma_{xy}=-54$ (T/Ω cm)/$\mu_0 H_z+2$ (1/T Ω cm)$\mu_0 H_z$. We have made similar measurements from $T=98$ to 70 K, and over this temperature range the form of the low field divergence in σ_{xy} varies from diverging slower than $1/H$ at high temperatures to diverging faster at our lowest temperatures. Samoilov *et al.*[23] have also measured σ_{xy} in the mixed state of Tl2212, where they fit σ_{xy} to $1/H$ at low fields.

Based on the anisotropic-mass model of Ref. 16, the anisotropy has the same effect as reducing the field component

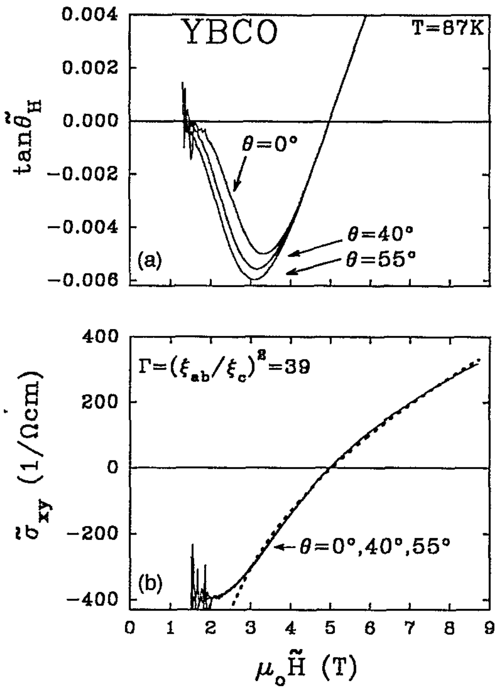

FIG. 3. (a) $\tan\tilde{\theta}_H=\tan\theta_H\tilde{H}/H_z$ vs scaled field $\tilde{H}=H\sqrt{\cos^2+\Gamma^{-1}\sin^2\theta}$ with $\Gamma=39$. (b) Scaled Hall conductivity $\tilde{\sigma}_{xy}=\sigma_{xy}\tilde{H}/H_z$ vs $\tilde{H}$ (solid curves). The dashed curve is a fit to the data with $\tilde{\sigma}_{xy}=-1426$ (T/Ω cm)/$\mu_0\tilde{H}+57$ (1/T Ω cm)$\mu_0\tilde{H}$.

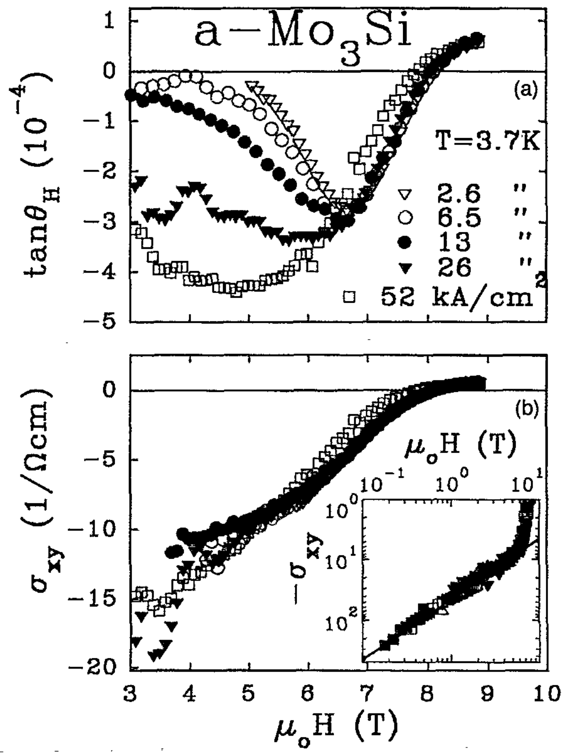

FIG. 4. (a) Nonohmic $\tan\theta_H$ vs H in an a-Mo_3Si film at $T=3.7$ K ($T_c\cong7.5$ K) for current densities $j=2.6$ to 52 kA/cm^2 all in excess of j_c. (b) Nonohmic σ_{xy} vs H, over the same range of currents as above. (Inset) $-\sigma_{xy}$ vs H on a log-log scale for the same range of currents. The solid curve has slope of -1.

in the superconducting planes. Thus in the limit of very large anisotropy, such as in NCCO and Tl2212, scaling behavior with $H_z=H\cos\theta$ is expected. YBCO, which is closer to being isotropic than NCCO or Tl2212, has a less obvious scaling behavior. The more general result of Ref. 16 says that the field should be scaled to $\tilde{H}=H\sqrt{\cos^2\theta+\Gamma^{-1}\sin^2\theta}$, and that there is a scaling factor $H_z/\tilde{H}$ for the Hall components of the conductivity and resistivity tensors, where again Γ is the effective mass anisotropy ratio. In Fig. 3(a) we plot $\tan\tilde{\theta}_H=\tan\theta_H\tilde{H}/H_z$ vs $\tilde{H}$ for $\theta=0°$, 40°, and 55° at $T=87$ K, and $\Gamma=39$ for our YBCO film. From the figure it is clear that the Hall angle increases as the field is tilted away from the c axis, similar to the angular dependence of ρ_{xy} in YBCO.[6] The enhancement may be due to a weakening of pinning by defects, as it has been argued that $\tan\theta_H$, and similarly ρ_{xx} and ρ_{xy}, depend on pinning,[20,21] and pinning by defects is anisotropic, being weaker when the field is directed into the ab plane.[16] The enhancement vanishes as $H\rightarrow H_{c2}$, presumably due to less flux pinning and a larger quasiparticle contribution to the Hall effect at high fields. In contrast, the enhancement is difficult to observe in NCCO and Tl2212, where, as a result of their large anisotropies the extrinsic pinning will be noticeably weaker only when the field is directed nearly parallel to the plane ($\theta\approx90°$). Unfortunately, within the field limit of our magnet (9 T) the signals are too small at these angles to observe this effect.

To understand how vortex motion is affected by pinning we consider the work of Vinokur, Geshkenbein, Feigel'man, and Blatter (VGFB),[20] who have argued that macroscopically the appropriate equation of motion for each vortex is

$$\eta\mathbf{v}+\alpha\mathbf{v}\times\hat{\mathbf{n}}=\phi_0\mathbf{j}\times\hat{\mathbf{n}}+\langle\mathbf{F}_{\rm pin}\rangle, \qquad (1)$$

where η is the viscous-drag coefficient associated with dissipation in the mixed state, $\mathbf{j}$ is the transport current density, $\hat{\mathbf{n}}$ is a unit vector in the direction of the magnetic field, $\mathbf{v}$ is the average vortex velocity, α determines the sign and the magnitude of the Hall angle in the absence of pinning by the relation $\tan\theta_H=\alpha/\eta$, $\phi_0=h/2e$ is the flux quantum, and $\langle\mathbf{F}_{\rm pin}\rangle$ is the average pinning force. VGFB further argue that the average pinning force can be replaced by a term which, to leading order, is linear in the vortex velocity,

$$[\eta+\gamma(v)]\mathbf{v}+\alpha\mathbf{v}\times\hat{\mathbf{n}}=\phi_0\mathbf{j}\times\hat{\mathbf{n}}, \qquad (2)$$

so that pinning has the effect of renormalizing the drag coefficient, $\eta\rightarrow\eta+\gamma(v)$. Therefore, $\tan\theta_H=\alpha/\eta\rightarrow\alpha/[\eta+\gamma(v)]$ will change as pinning is varied. The Hall conductivity $\sigma_{xy}=\alpha/\phi_0B$ is predicted to be *independent of pinning.*

In Fig. 3(b) we plot the scaled Hall conductivity $\tilde{\sigma}_{xy}=\sigma_{xy}\tilde{H}/H_z$ vs $\tilde{H}$. As can be seen from the figure the data for all θ collapse to a single curve using one adjustable parameter $\Gamma=39$ ($\xi_{ab}\cong6\xi_c$, in agreement with other measurements[24]). Similar results have been previously reported by Harris *et al.*[7] In light of the behavior of $\tan\tilde{\theta}_H$ in Fig. 3(a), the scaling of the Hall conductivity is remarkable, and adds credence to the idea that σ_{xy} does not depend on disorder.[20,21] The dashed curve is given by $\tilde{\sigma}_{xy}=-1426$ (T/Ω cm)/$\mu_0\tilde{H}+57$ (1/T Ω cm)$\mu_0\tilde{H}$, similar to the behavior we observe in NCCO and Tl2212. In all our thin-film YBCO samples we observe the deviation of the data from the low end of the dashed curve, i.e., deviation from $1/H$. We observe similar results closer to T_c, as well as down to our lowest temperatures, $T=83$ K. At low fields the pinning is strongest, thus these results may signal that σ_{xy} has some disorder dependence in the strong-pinning limit.[25] More work is in progress to resolve this issue.

Next we consider our high-current measurements on low-pinning a-Mo_3Si as an alternative probe to test the disorder independence of σ_{xy}. The most direct way to verify the predictions for $\tan\theta_H$ and σ_{xy} within the theoretical framework of VGFB is with high currents. The increase of the transport-current density $\mathbf{j}$ drives the vortices to higher velocities $\mathbf{v}$ via the Lorentz force $\phi_0\mathbf{j}\times\hat{\mathbf{n}}$, thus reducing the pinning parameter $\gamma(v)$ ($\propto 1/\sqrt{v}$, for example[20]). a-Mo_3Si is ideal for this purpose because of its very small depinning current density ($j_c\approx 1000$ A/cm^2 at $T=4.2$ K and 1 T$<\mu_0 H<4$ T), which is easily exceeded with minimal sample heating ($\leqslant 15$ mW/cm^2). At high fields where the sample resistance is significantly larger we use a pulsed (20 μs) current technique to further avoid heating. In Fig. 4(a) we plot $\tan\theta_H$ vs H at $T=3.7$ K for a range of current densities all in excess of j_c (in the nonohmic regime). We observe a large increase in the Hall angle with increasing current density, similar to that observed in YBCO as the field is rotated into the plane (Fig. 3). Another similarity between $\tan\theta_H$ in Figs. 3 and 4 is the vanishing of the enhancement as $H\rightarrow H_{c2}$.

In Fig. 4(b) we show σ_{xy} vs H at the same current densities as in Fig. 4(a). Remarkably, σ_{xy} is independent of the current density, an excellent confirmation of the pinning independence of σ_{xy}. This result casts serious doubt on theories which invoke pinning to explain the sign anomaly.[5] (The deviation at $j=52$ kA/cm^2 is a result of j approaching the depairing critical current j_0, so T_c is slightly suppressed.) The log-log scale in the inset indicates that over more than a decade of our lowest fields $\sigma_{xy}\sim 1/H$ (the solid curve has slope -1), and extends to fields nearly two orders of magnitude smaller than $\mu_0 H_{c2}\approx 7.5$ T. Our measurements at both lower and higher temperatures ($T=7$ and 1.4 K) are consistent with these results.

In summary, we have shown that the angular dependence of the Hall conductivity for the anisotropic superconductors NCCO, Tl2212, and YBCO can be scaled in terms of their anisotropy. This is possible, in particular for YBCO, because the effects of pinning drop out of σ_{xy}. We further demonstrated the pinning independence of σ_{xy} using high-current densities on a-Mo_3Si. The implications of this result are quite important as the Hall conductivity presents experimenters with a measurable transport quantity which is not clouded by extrinsic pinning effects. This result, in particular, contradicts theories which explain the sign anomaly as a pinning effect.[5] Finally, our data provides evidence for a universal low-field behavior of $\sigma_{xy}\sim 1/H$, as predicted for vortex dynamics.[4,8,9]

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